Revealed-Preference Analysis with Framing Effects

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In many settings, decision makers' behavior is observed to vary on the basis of seemingly arbitrary factors. Such framing effects cast doubt on the welfare conclusions drawn from revealed-preference analysis. We relax the assumptions underlying that approach to accommodate settings in which framing effects are present. Plausible restrictions of varying strength permit either partial or point identification of preferences for the decision makers who choose consistently across frames. Recovering population preferences requires understanding the empirical relationship between decision makers' preferences and their sensitivity to the frame. We develop tools for studying this relationship and illustrate them with data on automatic enrollment into pension plans.

Introduction

In many settings, people's choices depend on seemingly arbitrary features of the decision-making environment, such as which option is the

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default, the order in which options are presented, or which features of the decision are salient. Such *framing effects* cast doubt on the welfare conclusions from revealed-preference analysis. For example, suppose that an internet company adopts an opt-out data policy, under which it can collect a customer's data to personalize advertising content unless the customer indicates otherwise. Prior research suggests that switching to an opt-in policy, under which customers must give permission before the company can collect their data, would reduce the number of customers who allow the company to do so (Johnson, Bellman, and Lohse 2002). Suppose that 40% of customers choose to allow data collection when the policy is opt-in and 65% do so when the policy is opt-out. Both policies let customers control the use of their data, but the choices observed under the two policies imply different conclusions about what customers prefer.

In this paper, we relax the assumptions underlying revealed-preference analysis to accommodate choice data contaminated by framing effects. We focus on binary decisions in which the choices of some decision makers vary according to a preference-irrelevant feature of the environment, which we refer to as a *frame* (Salant and Rubinstein 2008). Examples of frames might include (1) which option is presented as the default, (2) the order in which options are displayed, (3) the reference point from which an option is evaluated, (4) whether the menu of options includes an irrelevant alternative, (5) the point in time at which a decision is made, and (6) which features of the available options are made salient. We assume that when decision makers choose consistently across frames, those choices reflect their preferences.[1]

Within this framework, we derive conditions for identifying preferences of various groups of decision makers. First, we show that when a frame pulls the choices of all decision makers in a uniform direction (frame monotonicity), one can identify the distribution of preferences for the consistent decision makers—that is, the decision makers unaffected by the framing effect. This is true even when each decision maker is observed making only one decision and observers lack ex ante knowledge about which decision makers are consistent. Under frame monotonicity, a decision maker who chooses "against the frame"—for example,

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[1] By "preferences," we mean the relative degree to which the available options further a decision maker's objectives, whatever those may be. Preferences are not defined according to observed choices; doing so would assume away the possibility of framing effects.

someone who chooses the option that is not the default—is consistent and prefers the option that she chooses. This fact, along with an exogeneity assumption concerning the assignment of decision makers to frames, allows us to point-identify the preferences of the consistent decision makers. Without frame monotonicity, the preferences of this group are partially identified, and we derive the corresponding bounds.

Next, we turn to the problem of identifying preferences for the full population of decision makers. Our key insight is that this problem shares important features with the classic selection-into-treatment problem from the program-evaluation literature. That is, once we have identified preferences for the subgroup of decision makers who are consistent, we can account for selection into that subgroup to recover preferences for the overall population. Stated this way, the transformed problem is both more familiar and more tractable than the original: economists have developed a range of tools for dealing with endogeneity problems of this sort, and we adapt several to our setting.

The first approach we develop is to extrapolate the preferences of the consistent decision makers to the inconsistent decision makers by adjusting for observable differences between the two groups. Recovering population preferences requires that consistency and preferences be uncorrelated conditional on these observables. As in other settings where researchers rely on matching estimators, the plausibility of this assumption depends on the nature of selection and what information about decision makers the researcher can observe.

Second, we develop decision-quality instruments, which exploit variation in decision makers' susceptibility to a frame but do not affect decision makers' preferences. For example, decision makers who are experimentally manipulated to have a higher shadow value of leisure (e.g., by facing greater time pressure) may be more likely to choose according to the frame, but such a manipulation is unlikely to affect which option they actually prefer. Decision-quality instruments identify the distribution of preferences for those decision makers whose susceptibility to the frame they affect. We develop techniques to extrapolate the preferences of this subgroup to the population.

Finally, we derive bounds on population preferences based on the consistent decision makers. The usefulness of the bounds depends on the strength of the frame—the bounds are tighter when more decision makers are consistent. One surprising finding from this analysis is that absent frame monotonicity, it may be that a majority of the population prefers one option even though a majority selects the other option under every frame that is observed.

A growing literature confronts the problem of preference identification in settings with framing effects. One proposal is to restrict preference inferences to the subset of observed choices in which a given decision

maker chooses consistently (Bernheim and Rangel 2009). In practice, however, individual decision makers are typically observed choosing under only one frame, which makes it difficult to detect which choices are consistent. Worse, this approach yields no information on the inconsistent decision makers—the very group whose behavior is shaped by the choice of frame. Further "refinements" can provide a path forward if the researcher can observe choices in a frame in which all decision makers are known to select their most preferred option (Chetty, Looney, and Kroft 2009; Allcott and Taubinsky 2015), but in many applications, such as those in which behavior is sensitive to defaults or ordering effects, there is little reason to believe that any observed frame satisfies this condition.

A different solution is to rely on a positive model of behavior that fully specifies the mapping from decision makers' preferences to their (potentially suboptimal) behavior (Rubinstein and Salant 2012). Inverting the model allows one to recover preferences from the decision makers' observed choices. However, in many cases the resulting welfare conclusions are sensitive to the researcher's choice between competing positive models that are difficult to distinguish observationally. In some cases, even a fully specified behavioral model is insufficient to recover preferences from choice data (Benkert and Netzer 2018).

We contribute to this literature by developing a framework for preference identification that strikes a middle ground between these approaches. Relative to Bernheim and Rangel (2009), our approach requires additional structure, but the payoff to that additional structure is significant: one can apply our results to the realistic class of settings in which individual decision makers are observed under only one frame and in which the researcher is not confident that any one of the observed frames induces all decision makers to choose optimally. Relative to imposing a specific behavioral model, an advantage of our framework is its generality: our central behavioral assumptions—that consistent decision makers choose optimally and frame monotonicity—hold under a wide range of models for why framing effects occur. Consequently, our approach can recover the preferences of the consistent decision makers, as well as bounds on population preferences, while remaining reasonably agnostic about the precise underlying model that generates the observed framing effect. In contrast, point-identifying population preferences requires pinning down the relationship between preferences and consistency and, implicitly, restricting the behavioral model that governs which decision makers are sensitive to the frame. We develop empirical tools to shed light on this relationship under a range of assumptions about the available data and the underlying behavioral model. Finally, our framework complements model-based approaches by making transparent the role the model's assumptions play in identification: within a broad class of models, distinguishing between behavioral and functional form assumptions matters

only to the extent that the assumptions offer conflicting predictions for the relationship between preferences and consistency.[2]

We illustrate our framework using data on participation in an employer-sponsored pension plan under two different default enrollment regimes. We show how the preference information that can be recovered from the data depends on the strength of the assumptions the researcher is willing to impose. Under relatively weak assumptions, we find that a sizable majority of the consistent employees prefer enrollment. We also document a strong positive relationship between employees' sensitivity to framing effects (opt-in vs. opt-out enrollment) and their preferences for enrollment in the plan. We conclude that under plausible assumptions, the data imply that a majority of employees prefer participation in the pension plan but that there is significant heterogeneity. For example, employees who are likely to leave the firm within 2 years disproportionately prefer not to participate.

Focusing on binary choices and binary frames highlights the identification challenge through the lens of the potential outcomes framework commonly used in the program-evaluation literature (Angrist, Imbens, and Rubin 1996),[3] but the intuition we develop is useful outside of this setting as well, as we illustrate in an extension. We also describe how the preference information identified by our approach can be combined with price variation to estimate traditional measures of cardinal welfare.

The paper proceeds as follows. Section I presents our notation and main assumptions. Section II presents results relating to the preferences of the consistent decision makers. Section III presents results relating to the full population. Section IV illustrates our identification results, using data on defaults and enrollment into employer-provided pension plans. Section V describes how the preference information we recover can be combined with price variation to derive conventional measures of cardinal welfare. The appendix (available online) contains proofs of propositions; derivations of standard errors; additional results relating to decision-quality instrument extrapolation; supplementary material relating to the empirical application; and generalizations relating to nonbinary

[2] We also contribute to a strand of literature that uses the preferences of a reference group of decision makers (whose choices are assumed to be optimal) as a guide to the rest of the population. In previous work, the reference group consists of experts, identified on the basis of information about experience, occupation, or familiarity with the subject matter (Bronnenberg et al. 2015; Handel and Kolstad 2015; Johnson and Rehavi 2016). Our contribution is to develop a method of applying this approach to settings characterized by framing effects, in which no ex ante information is available to identify members of the reference group. Rather, inclusion in the reference group (composed of the consistent decision makers) emerges endogenously from observable decision-making behavior.

[3] Unlike other applications of the potential outcomes framework of which we are aware, our goal is not to identify the causal effects of one variable on another but rather to remove variation in observed choices due to framing effects, isolating the variation due to preferences.

frames, nonbinary menus, and settings in which decision makers' assignment to frames is nonrandom.

I. Formal Framework

A. Notation and Assumptions

Each decision maker i is observed to choose from a fixed menu $\mathbf{S} = \{0, 1\}$ under one of two possible frames $D_i \in \{0, 1\}$.[4] Let $Y_i(0)$ and $Y_i(1)$ denote what i would choose under frames $D_i = 0$ and $D_i = 1$, respectively. Decision makers have strict ordinal preferences over the available options, with the most preferred option denoted by $Y_i^* \in \{0, 1\}$. Each decision maker is characterized by a vector of random variables $(Y_i(0), Y_i(1), D_i, Y_i^*)$, drawn from some underlying population distribution. For each i, the researcher observes the pair (Y_i, D_i), where $Y_i = Y_i(1)D_i + Y_i(0)(1 - D_i)$. The researcher does not observe Y_i^* and observes only one of $Y_i(0)$ and $Y_i(1)$, depending on the frame D_i.

We denote mean choices among decision makers assigned to each frame by $\bar{Y}(1) \equiv E[Y_i|D_i = 1]$ and $\bar{Y}(0) \equiv E[Y_i|D_i = 0]$. Unless otherwise noted, the operator $E[\cdot]$ denotes an expectation over the population distribution of the random variable inside the square brackets (which may be a function of primitive random variables). We assume for exposition that these population moments are directly observable to the researcher, deferring issues of finite-sample statistical inference to the appendix. Without loss of generality, $\bar{Y}(1) \geq \bar{Y}(0)$. To illustrate the notation with the privacy example from the introduction, let Y_i indicate whether i allows a company to use her data, $D_i = 1$ indicate the opt-out regime, and $D_i = 0$ indicate the opt-in regime, so that $\bar{Y}(1) = 0.65$ and $\bar{Y}(0) = 0.40$.

Decision makers either choose consistently or choose in a way that is sensitive to the frame. We denote consistency by $C_i = \mathbb{I}\{Y_i(0) = Y_i(1)\}$. Because each decision maker is observed under one frame, C_i is not observed. We assume that the fraction of consistent decision makers is strictly positive, $E[C_i] > 0$.

Four assumptions (A1–A4) form the core of our analysis.

Assumption A1 (Frame separability). For all i, Y_i^* does not depend on D.

Frame separability is an assumption about the content of decision makers' preferences. It is implicit in the above notation because we do not index Y_i^* by D_i—otherwise, individuals would have two random variables for Y_i^* (one for each frame). Assumption A1's role is to define which features of the decision-making environment are treated as a

[4] Because the menu is constant across decision makers, our notation conditions on it implicitly. The appendix considers generalizations to nonbinary menus and nonbinary frames.

frame.[5] Features of a decision that affect choice but are relevant to decision makers' preferences over the available options are not frames. For example, if a decision maker chooses hot chocolate from {hot chocolate, ice cream} under $D = 0$ and ice cream from {hot chocolate, ice cream} under $D = 1$, there would be no framing effect if D indicates whether the season is winter or summer. Importantly, frame separability does not require decision makers to be irrational; a feature of the environment that imposes a cognitive cost for selecting one of the options would constitute a frame, as long as it did not also affect decision makers' preference for ending up with one option or the other.[6]

The remaining assumptions concern the distribution of $(Y_i(0), Y_i(1), D_i, Y_i^*)$ in the population.

ASSUMPTION A2 (Frame exogeneity). $(Y_i(0), Y_i(1), Y_i^*) \perp D_i$.

Frame exogeneity is an assumption about the data-generating process by which decision makers are assigned to frames. The assumption ensures that differences in observed choices under different frames are due to the effect of the frames, rather than to differences in the composition of decision makers observed under each frame. Frame exogeneity is guaranteed when decision makers are randomly assigned to frames.[7]

We now turn to the link between choices and preferences. The standard revealed-preferences approach is to infer decision makers' preferences directly from their choices.

RPA (Revealed-preferences assumption). For all i, $Y_i^* = Y_i$.

In our setting, a framing effect occurs when assumptions A1 and A2 are satisfied and one observes $\bar{Y}(1) \neq \bar{Y}(0)$. By definition, framing effects violate the RPA. This is because assumption A2 implies $\bar{Y}(0) = E[Y_i(0)]$ and $\bar{Y}(1) = E[Y_i(1)]$, and $E[Y_i(0)] \neq E[Y_i(1)]$ implies $Y_i(1) \neq Y_i(0)$ for some subset of decision makers. But for any such decision maker, assumption A1 and the RPA imply $Y_i(1) = Y_i^* = Y_i(0)$, yielding a contradiction. The next assumption weakens the RPA to accommodate choice data in which framing effects are present.

ASSUMPTION A3 (Consistency principle). For all i, $C_i = 1 \Rightarrow Y_i = Y_i^*$.

Under the consistency principle, preferences are guaranteed to be revealed by choices only for decision makers who choose consistently across frames. Because consistency is a function of $Y_i(0)$ and $Y_i(1)$, the assumption constrains the joint distributions of $Y_i(0)$, $Y_i(1)$, and Y_i^*. The consistency principle weakens the RPA by the minimum necessary to accommodate an apparent framing effect. In the online-privacy example

[5] This assumption is explicit in Salant and Rubinstein (2008) and implicit in Bernheim and Rangel (2009), who require it for determining when two potentially conflicting choice situations differ in terms of the frame or in terms of the available menu items. In this sense, frame separability is the property that distinguishes variation in frames from variation in menu items.

[6] For a discussion of related issues, see Bernheim and Taubinsky (2018).

[7] The appendix considers several generalizations in which frame exogeneity is relaxed.

described above, the assumption implies that a customer who would choose to keep her data private under both the opt-in and opt-out frames does in fact prefer that her data be kept private. Like the RPA, the assumption fails when decision makers suffer from biases that cause them to make the same mistake under every frame in which they are observed.

The following monotonicity assumption permits us to recover aggregate information about consistency even though C_i is not observable at the individual level.

ASSUMPTION A4 (Frame monotonicity). For all i, $Y_i(1) \geq Y_i(0)$.

Frame monotonicity requires that when a frame affects choice, it does so in the same direction for each affected decision maker. It thus constrains the joint distribution of $Y_i(0)$ and $Y_i(1)$. In the online-privacy example, frame monotonicity fails if some customers choose to allow access to their data if and only if doing so is not the default. Much of our discussion will assume frame monotonicity, but we also derive partial identification results for settings in which it fails.

B. *Examples of Framing Effects*

The following are examples of behavioral models that might generate a particular observed framing effect.

1. Example 1: Default Effects

Our running example will concern default effects. The frame, $D \in \mathbf{S}$, encodes which option is the default. Decision makers choose according to their (fixed) preferences $u_i(Y)$ over a subset $\mathbf{\Gamma}_i \subseteq \mathbf{S}$ of options that they consider (as in Masatlioglu, Nakajima, and Ozbay 2012). Decision makers can be either active or passive. When active, decision makers consider both options, $\mathbf{\Gamma}_i(D) = \mathbf{S}$. When passive, they consider only the option that is the default, $\mathbf{\Gamma}_i(D) = \mathcal{D}$. Because u_i does not depend on the default, frame separability is satisfied. To see that the consistency principle holds, suppose that some individual i chooses the same option under both defaults, $Y_i(0) = Y_i(1) = 1$. Because i chooses 1 under $D = 0$, we know that $1 \in \mathbf{\Gamma}_i(0)$ and therefore that i is active under $D = 0$, with $u_i(1) > u_i(0)$. To see that frame monotonicity holds, suppose that $Y_i(0) = 1$ (when $Y_i(0) = 0$, the condition holds trivially). As above, $Y_i(0) = 1$ implies that i is active under $D = 0$ and prefers option 1. Because $1 \in \mathbf{\Gamma}_i(1)$ as well, we know that $Y_i(1) = 1$. Hence, $Y_i(1) \geq Y_i(0)$.

2. Example 2: Time Inconsistency

At date t, decision makers choose between receiving some amount y_0 at date $t + k$ or some other amount $y_1 > y_0$ at date $t + k + 1$. As in Laibson

(1997), individuals choose according to the following behavioral utility function: $\tilde{u}_{it} = u(z_t) + \beta_i \Sigma_{k=1}^{\infty} \delta_i^t u(z_{t+k})$, where z_t is total income at time t, $\beta_i \leq 1$, and $\delta_i^t < 1 \ \forall \ t$. The frame denotes whether $k = 0$ ($D = 0$) or $k > 0$ ($D = 1$). Assuming that the amounts in y_0 and y_1 are small relative to background income, it is straightforward to show that when $D = 0$, i chooses y_1 iff $y_0/y_1 < \beta_i \delta_i$ and that when $D = 1$, i chooses y_1 iff $y_0/y_1 < \delta_i$. Welfare can be evaluated according to either $\beta_i \delta_i^t$ (the short-term view) or δ_i^t (the long-term view; Bernheim 2009). In either case, it is straightforward to verify that the consistency principle holds and that frame monotonicity holds as long as $\beta_i \leq 1$ for all i. In addition, frame separability holds as long as the welfare-relevant discount rate does not vary within an individual on the basis of the time the decision is made (as it would under $\tilde{u}_{it}$).

3. Example 3: Bias Unrelated to Observed Framing Effect

Consider a setting that involves both default effects and present bias. As above, decision makers choose between y_0 at date $t + k$ and $y_1 > y_0$ at date $t + k + 1$, but now either y_0 or y_1 is set to be the default. Welfare is given by $u_{it} = u(z_t) + \Sigma_{k=1}^{\infty} \delta_i^t u(z_{t+k})$. As in example 1, some decision makers are passive and choose whichever option is set as the default. The other decision makers are active (but present biased) and choose according to $\tilde{u}_{it} = u(z_t) + \beta_i \Sigma_{k=1}^{\infty} \delta_i^t u(z_{t+k})$. Choices are observed only at $k = 0$. The consistency principle fails here because the decision makers who are consistent with respect to the default are present biased: an active decision maker chooses y_1 if $y_0/y_1 < \beta_i \delta_i$ but prefers y_1 if $y_0/y_1 < \delta_i$. Hence, some of those who consistently select y_0 would actually prefer y_1. This example highlights that our framework can recover preferences only when any mistakes are due to an observed framing effect. In settings where a bias is present but no inconsistency is observed, our approach (like traditional revealed-preference analysis) would incorrectly infer preferences from consistent choices. If choices were observed under each frame and also under both $k = 0$ and $k > 0$, one could apply our approach to first eliminate the framing effect at each value of k and then to use those results to estimate the behavioral parameters β and δ.

II. Identifying Consistent Preferences

We initially focus on the consistent decision makers—that is, those whose behavior is not affected by the frame. Recovering the preferences of this group would be trivial if decision makers were observed under both frames; in that case, an observer could identify which decision makers were consistent and, using the consistency principle, which options

the consistent decision makers preferred. However, many real-world data sets do not have this property, and even when they do, the order in which decision makers are exposed to frames may itself affect behavior (LeBoeuf and Shafir 2003). The following proposition provides conditions for the identification of consistent decision makers' preferences when each decision maker is observed under a single frame.

PROPOSITION 1. Let $\bar{Y}_C \equiv \bar{Y}(0)/(\bar{Y}(0) + 1 - \bar{Y}(1))$.

1.1. Under assumptions A1–A4, $E[Y_i^*|C_i = 1] = \bar{Y}_C$.
1.2. Under assumptions A1–A3, $\bar{Y}_C \geq 1/2 \Rightarrow \bar{Y}_C \leq E[Y_i^*|C_i = 1] \leq 1$, and $\bar{Y}_C \leq 1/2 \Rightarrow 0 \leq E[Y_i^*|C_i = 1] \leq \bar{Y}_C$. These bounds are sharp under the stated assumptions.

Proposition 1.1 follows from the insight that, under frame monotonicity, only consistent decision makers choose against the frame (i.e., they choose $Y_i(0) = 1$ or $Y_i(1) = 0$). Frame exogeneity guarantees that the assignment of individuals to frames is uncorrelated with preferences or consistency, which means that we can treat the set of decision makers choosing against the frame as a representative sample of all consistent choosers. In turn, the consistency principle ensures that the observed choices of this group reveal this group's preferences. As a result, the denominator of $\bar{Y}_C$ measures the fraction of decision makers who are consistent and the numerator measures the subset of that group with $Y_i^* = 1$. A formal proof of proposition 1.1, and of further results, is contained in the appendix.

Proposition 1.2 provides a partial identification result that is robust to failures of frame monotonicity. Borrowing terminology from Angrist, Imbens, and Rubin (1996), define *frame defiers* as the subset of inconsistent decision makers who select $Y_i(0) = 1$ and $Y_i(1) = 0$. Frame defiers would be misclassified as consistent by the logic underlying proposition 1.1. To understand the intuition behind the proof, note that a decision maker who chooses against a frame may be either consistent or a frame defier. Because frame defiers are assigned to the two different frames in equal proportions (by frame exogeneity), proposition 1.1 will classify half of the frame defiers as consistently choosing $Y_i = 1$ and half as consistently choosing $Y_i = 0$. Ignoring the presence of frame defiers therefore biases $\bar{Y}_C$ toward 1/2.

The reasoning behind proposition 1 is further illustrated in table 1, which applies the result to the online-privacy example from the introduction.

The preference information recovered by proposition 1 is important for several reasons. First, if one's philosophical starting point is that inconsistent decision makers lack normatively relevant preferences (see Fischhoff 1991), then proposition 1 is the end point of the analysis; it isolates the normatively relevant parameter from the noise induced by the

TABLE 1
ILLUSTRATION OF PROPOSITION 1

	Choose Not to Enroll, Opt-In Regime, $Y_i(0) = 0$	Choose to Enroll, Opt-In Regime, $Y_i(0) = 1$
Choose not to enroll, opt-out regime, $Y_i(1) = 0$	.35	.00
Choose not to enroll, opt-out regime, $Y_i(1) = 1$	.25	.40
Fraction consistent	$E[C_i] = 0.4 + 0.35 = 0.75$	
Fraction of consistent decision makers preferring option 1	$E[Y_i^* \mid C_i = 1] = \bar{Y}_C = 0.40/(0.4 + 0.35) \approx 0.53$	
Bounds on consistent preferences, without assumption A4	$0.53 \leq E[Y_i^* \mid C_i = 1] \leq 1$	

NOTE.—Under frame monotonicity, top-right quadrant = 0; top-left quadrant = $1 - \bar{Y}(1)$; bottom-right quadrant = $\bar{Y}(0)$; bottom-left quadrant = $\bar{Y}(1) - \bar{Y}(0)$. $\bar{Y}_C$ is biased toward 1/2, as frame defiers are equally assigned to the top-left and bottom-right quadrants by frame exogeneity (by definition, they cannot be in the bottom-left quadrant).

frames.[8] Second, when population preferences are known—what Bernheim and Rangel (2009) refer to as a "refinement"—proposition 1 can be used in conjunction with that information to recover the preferences of the inconsistent decision makers.[9] The preferences of this group can be an input into welfare calculations (see sec. V) but are not directly revealed under a refinement. Finally, the preferences of the consistent decision makers may be used to recover the preferences of the remainder of the population by accounting for selection into the consistent subpopulation, which is our focus in the next section.

III. Identifying Population Preferences

This section develops several methods for using the preferences of the consistent decision makers to shed light on the rest of the population. To frame the problem, one can use the law of iterated expectations to write

$$E[Y_i^*] = E[C_i]E[Y_i^* \mid C_i = 1] + (1 - E[C_i])E[Y_i^* \mid C_i = 0]. \quad (1)$$

From proposition 1.1, $E[C_i]$ and $E[Y_i^* \mid C_i = 1]$ are identified under assumptions A1–A4, but $E[Y_i^* \mid C_i = 0]$ is entirely unrestricted. Consequently, the formal challenge in recovering $E[Y_i^*]$ is the same as the

[8] For example, suppose that the fraction of voters supporting a referendum varies with the question wording. It is straightforward to show that randomizing the wording evenly across two options biases the average vote share toward 0.5, which can affect the outcome if the referendum requires a supermajority to pass. In cases like this, isolating the average choices of the consistent voters may be the best option.

[9] Formally, when $E[Y_i^*]$ is known, the law of iterated expectations allows us to recover $E[Y_i^* \mid C_i = 0] = (E[Y_i^*] - E[Y_i^* \mid C_i = 1]E[C_i])/(1 - E[C_i])$.

standard sample-selection problem that arises in the program-evaluation literature (Manski 1989).[10] Specifically, when selection into the consistent subpopulation is nonrandom, the preferences of that group can yield a biased estimate for the preferences of the population:[11]

$$E[Y_i^*] = E[Y_i^* | C_i = 1] - \frac{1}{E[C_i]} \text{Cov}(Y_i^*, C_i). \tag{2}$$

However, when susceptibility to the frame is uncorrelated with preferences—a condition we refer to as *consistency independence*—equation (2) highlights that $E[Y_i^*] = E[Y_i^* | C_i = 1]$. In that case, proposition 1 permits identification of population preferences without further adjustments. More generally, equation (2) shows that under assumptions A1–A4, recovering the covariance between preferences and consistency is equivalent to identifying the ordinal preferences of the population; the particulars of the behavioral model matter only to the extent that they shape this relationship.

The nature of the selection in equations (1) and (2) depends on the model determining which agents are consistent. To illustrate, consider the following two potential models for default effects, both of which are nested by the default effects example in section I.

1. *Decision-Making Types Model of Default Effects*

Suppose that sensitivity to the default is an innate characteristic or influenced by factors such as education or prior experience that are exogenous to the specific choice being considered (as in Chetty et al. 2014). The distribution of active types depends on the following statistical model: $C_i = 1 \Leftrightarrow \tilde{C}_i \geq 0$, $\tilde{C}_i = \beta^C \theta_i^C + \eta_i^C$, where θ_i^C denotes the vector of individual characteristics that determine whether one is active and η_i^C denotes idiosyncratic variation across individuals. Similarly, $Y_i^* = 1 \Leftrightarrow \tilde{Y}_i \geq 0$, $\tilde{Y}_i = \beta^Y \theta_i^Y + \eta_i^Y$, where θ_i^Y denotes the vector of characteristics that determine ordinal preference and η_i^Y denotes idiosyncratic variation.[12] Assume that η_i^C and η_i^Y are independent of the other random variables in the model and of each other. We then have $\text{Cov}(Y_i^*, C_i) = p(\tilde{Y}_i > 0; \tilde{C}_i > 0) - p(\tilde{Y}_i > 0)p(\tilde{C}_i > 0)$. Consistency independence fails if θ_i^C and θ_i^Y contain common characteristics or characteristics that are correlated in the population of decision makers.

[10] One important difference is that in the typical sample-selection context, the researcher can identify which units have been selected into the sample and which have not. In contrast, consistency is unobservable in our setup.

[11] Equation (2) follows from the definitions of covariance and conditional expectation.

[12] Note that some of the characteristics contained in θ_i^Y and θ_i^C are potentially unobservable.

2. *Bounded-Rationality Model of Default Effects*

Suppose instead that variation in consistency is driven in part by the utility stakes of the choice at hand. Decision makers must incur a cognitive cost, $\gamma_i \geq 0$, to choose actively and consider an option that is not the default. Decision makers first choose whether to be active or passive and then choose from the options they consider. We assume that decision makers know which option is the default at the time they make both of these decisions. Let $\Delta u_i = u_i(1) - u_i(0)$, and let $F_\Delta(\cdot)$ denote its cumulative distribution over the population of decision makers. Consistency is determined by the net benefit to i of choosing her most preferred option when that option is not the default, $\tilde{C}_i = |\Delta u_i| - \gamma_i$, and we let $F_{\tilde{C}}(\cdot)$ describe its cumulative distribution. Decision makers are active iff $\tilde{C}_i > 0$, so the fraction of consistent decision makers in the population is given by $E[C_i] = 1 - F_{\tilde{C}}(0)$. Similarly, the fraction with $Y_i^* = 1$ is given by $E[Y_i^*] = 1 - F_\Delta(0)$. One can then derive the relationship between preferences and consistency as $\text{Cov}(Y_i^*, C_i) = (1 - F_\Delta(0))(F_{\tilde{C}}(0) - F_{\tilde{C}|\Delta u>0}(0))$.[13] Setting aside the trivial case in which preferences are uniform, consistency independence requires $F_{\tilde{C}}(0) = F_{\tilde{C}|\Delta u>0}(0)$. This would obtain, for instance, when the forgone utility from following a nonoptimal default is symmetric across agents with opposite ordinal preferences. Intuitively, the identification challenge in this model is that consistency depends on the "stakes" of the decision, and empirically, the stakes may differ among decision makers with different ordinal preferences. When this source of variation is unimportant relative to decision-maker characteristics in explaining consistency, as would be the case when the variance of $|\Delta u_i|$ is negligible relative to the variance in γ_i, this model approximates the decision-making types model described above (see also app. fig. 3).

Note that in this example we assume that the decision maker knows Δu_i with certainty (see also Conlisk 1996). One could alternatively suppose that the individual decides whether to choose actively on the basis of whether the expected net benefit of doing so exceeds the cost. As it is nested by the general model of default effects presented in example 1 of section I.B, such a model would not violate our core assumptions. The model imposes a similar set of challenges for extrapolation to population preferences as the model with certainty. We discuss additional nuance introduced by this type of model in appendix A.

These examples illustrate that consistency independence is likely to fail in many applications. The remainder of this section proposes a range of empirical methods for shedding light on the relationship between preferences and consistency.

[13] The derivation follows from the definition of covariance, $\text{Cov}(Y_i^*, C_i) = E[Y_i^* C_i] - E[Y_i^*]E[C_i]$, the expressions for $E[Y_i^*]$ and $E[C_i]$, and the fact that $E[Y_i^* C_i] = E[Y_i^*]E[C_i | Y_i^* = 1] = E[Y_i^*](1 - F_\Delta(0) | Y_i^* = 1)$.

A. *Partial Identification*

The following result clarifies the limits of what can be learned about population preferences without the imposition of additional behavioral assumptions.

PROPOSITION 2.

2.1. Under assumptions A1–A4, $E[Y_i^*] \in [\bar{Y}(0), \bar{Y}(1)]$.
2.2. Under assumptions A1–A3, $\max\{\bar{Y}(0) - (1 - \bar{Y}(1)), 0\} \leq E[Y^*] \leq \min\{\bar{Y}(0) + \bar{Y}(1), 1\}$. In addition, the bounds in propositions 2.1 and 2.2 are sharp under the stated assumptions.

The result in proposition 2.1 follows directly from the equivalence of the standard sample-selection problem and our setting, once the assumptions required for proposition 1 are imposed (Manski 1989). Intuitively, the fraction of the population that prefers an option lies between the fraction choosing that option under each of the two frames. As a result, the bounds will be relatively informative when the fraction of inconsistent decision makers is small.

Without frame monotonicity, we obtain weaker, one-directional bounds for population preferences. The result in proposition 2.2 follows from substituting the partial identification results in proposition 1.2 into equation (1). In particular, $E[Y_i^* | C_i = 1]$ and $E[C_i]$ can be identified, given information on the prevalence of frame defiers. Knowing that $E[Y_i^* | C_i = 1] \in [0, 1]$ constrains the prevalence of frame defiers, which then yields bounds on the value of $E[Y_i^*]$. The farther $\bar{Y}(0)$ is from $1 - \bar{Y}(1)$, the more informative the bounds will be.[14] Notably, when frame monotonicity fails, proposition 2.2 shows that it is possible that a majority of decision makers choose one option under both frames even though the other option is preferred by a majority of all decision makers.

B. *Adjusting for Observable Correlates of Consistency*

A classic approach to overcoming selection problems is to condition on observables. Such an approach is useful here when the correlation between preferences and consistency is driven by characteristics of decision makers that are observable to the researcher.

Formally, suppose that decision makers exhibit a vector of observable characteristics, denoted by random variable $X_i \in \mathbf{X}$. Define $\bar{Y}(D, X) = E[Y_i(D) | D_i = D, X_i = X]$ to be the cell-specific analogs to the population

[14] When $\bar{Y}(0) = 1 - \bar{Y}(1)$, the bounds are entirely uninformative because the data do not constrain the fraction of frame defiers, and as a result, we cannot rule out $E[C_i] = 0$. Consequently, when $\bar{Y}(0) = 1 - \bar{Y}(1)$, any $E[Y_i^*] \in [0, 1]$ is feasible under assumptions A1–A3.

means defined above. Define $q(X)$ as the ratio of consistent decision makers with $X_i = X$ relative to all consistent decision makers,

$$q(X) = \frac{\bar{Y}(0, X) + 1 - \bar{Y}(1, X)}{E_{X_i}[\bar{Y}(0, X_i) + 1 - \bar{Y}(1, X_i)]},$$

where $E_{X_i}[\cdot]$ is the expectation over the random variable X_i. We define $s(X)$ as the corresponding ratio for the inconsistent decision makers,

$$s(X) = \frac{\bar{Y}(1, X) - \bar{Y}(0, X)}{E_{X_i}[\bar{Y}(1, X_i) - \bar{Y}(0, X_i)]}.$$

Finally, we assume that frame exogeneity holds, conditional on each value of X.

ASSUMPTION A2′ (Conditional frame exogeneity). For all $X \in \mathbf{X}$, $(Y_i(1), Y_i(0)) \perp D_i | X_i = X$.

The identification strategy we propose in such settings is as follows: first, estimate the preferences of consistent decision makers with a given set of observable characteristics; second, extrapolate preferences from consistent to inconsistent decision makers with the same observable characteristics; and third, aggregate preferences across cells on the basis of estimated distributions of characteristics in the full population or the subpopulation of inconsistent decision makers. A barrier to employing this familiar approach in our context is that we cannot directly observe consistency. The following lemma, analogous to Abadie (2003), provides conditions under which the aggregate distribution of characteristics among the consistent and inconsistent decision makers can nonetheless be identified.

LEMMA 1. Under assumptions A1, A2′, A3, and A4,

1.1. for any X, $p(X_i = X | C_i = 1) = q(X)p(X_i = X)$;
1.2. for any X, $p(X_i = X | C_i = 0) = s(X)p(X_i = X)$.

The next step in the identification strategy proceeds under the following assumption.

ASSUMPTION A5 (Conditional consistency independence). For all individuals i and all observable characteristics $X \in \mathbf{X}$, $\mathrm{Cov}(Y_i^*, C_i | X_i = X) = 0$.

Conditional consistency independence requires that consistent and inconsistent decision makers with the same observable characteristics have the same distribution of preferences. The assumption is analogous to one commonly employed in the program-evaluation literature, that is, that observationally equivalent individuals do not sort on the unobserved gain to treatment (e.g., Angrist and Fernández-Val 2013). It is also similar to the type of assumption that has been relied on in the line of papers described in the introduction (e.g., Bronnenberg et al. 2015),

in which the preferences of a reference group of experts is extrapolated to the population.

Exploiting lemma 1, along with conditional consistency independence, the following proposition formalizes the matching-on-observables identification strategy described above.

PROPOSITION 3. Let $\bar{Y}_C(X) = \bar{Y}(0, X)/(\bar{Y}(0, X) + 1 - \bar{Y}(1, X))$. Under assumptions A1, A2′, A3, A4, and A5,

3.1. $E[Y_i^*] = E_X[\bar{Y}_C(X_i)]$;
3.2. $E[Y_i^*|C_i = 0] = E_X[s(X_i)\bar{Y}_C(X_i)]$.[15]

As with any matching-on-observables approach, the plausibility of this approach will depend on the detail and nature of the observable characteristics as well as the underlying positive model of behavior, as demonstrated by the following two examples.

1. Decision-Making Types Model of Default Effects

As above, suppose that preferences and consistency in the population are (respectively) characterized by latent index models $\tilde{Y}_i = \beta^Y\theta_i^Y + \eta_i^Y$ and $\tilde{C}_i = \beta^C\theta_i^C + \eta_i^C$, but that, unlike the case above, we now interpret θ_i^Y and θ_i^C to denote the vector of observable characteristics. Accordingly, η_i^Y and η_i^C denote the unobservable determinants of preferences and consistency. Conditional consistency independence requires that η_i^Y and η_i^C are independent, conditional on θ_i^Y and θ_i^C. For example, it could be that highly educated customers are less likely to prefer that companies use their personal data and are more likely to choose consistently across default regimes but that, conditional on education, preferences and consistency are uncorrelated.

2. Bounded-Rationality Model of Default Effects

Let $F_{\tilde{C}|X}(\cdot)$ denote the cumulative density of $\tilde{C}_i$ after conditioning on $X_i = X$. It is straightforward to show that conditional consistency independence requires $F_{\tilde{C}|X}(0) = F_{\tilde{C}|X,\Delta u>0}(0)$ for each X. This holds if the observed characteristics absorb enough variation in $|\Delta u_i|$ and γ_i such that the remaining, unobserved variation in consistency is uncorrelated with ordinal preferences. For example, suppose that the set of observables is rich enough to absorb variation in consistency associated with the utility

[15] Replacing assumption A2 with assumption A2′ in proposition 1.1 implies that $E[C_i] = E_X[\bar{Y}(0, X_i) + 1 - \bar{Y}(1, X_i)]$ and $E[Y^*|C_i = 1] = E_X[q(X_i)\bar{Y}_C(X_i)]$. Even when assumption A2 is satisfied, this revised estimator for $E[C_i]$ is preferable for applying proposition 3 in finite samples, as a result of possible spurious correlation between the observables and the frame.

stakes of the decision, $X_i = X_j \Rightarrow |\Delta u_i| = |\Delta u_j|$ for any two individuals i and j. Conditional consistency independence would then hold if the remaining variation in opt-out costs is uncorrelated with the remaining variation in ordinal preferences, a sufficient condition for which is that the structural parameters of the model are (conditionally) independently distributed, $(\gamma_i \perp \Delta u_i)|X_i = X$. In contrast, when the (conditional) variation in consistency is driven by selection on the gains to choosing actively, both consistency and ordinal preferences are driven by the same underlying structural parameter (Δu_i); hence, one would not expect their distributions to be independent, except in special cases.[16]

C. *Decision-Quality Instruments*

When conditioning on observables does not yield a credible identification strategy, researchers sometimes turn to instrumental variables designs. To develop the analog to that strategy here, we introduce the notion of a *decision-quality instrument*, a component of the decision-making environment that affects decision makers' consistency but that is unrelated to their preferences.[17]

Formally, the decision-quality instrument is a new random variable, $Z_i \in \{0, 1\}$. Choices can depend on both the frame and the instrument, $Y_i(D, Z)$, so there are now four potential outcomes. Each decision maker chooses once under a single (D, Z) combination. For each i, we observe (Y_i, D_i, Z_i), where $Y_i = Y_i(D_i, Z_i)$. Consistency is defined at each value of the instrument, $C_i(Z) = \mathbb{I}\{Y_i(0, Z) = Y_i(1, Z)\}$. We denote the fraction of decision makers choosing $Y_i = 1$ under a given (D, Z) combination by $\bar{Y}(D, Z) \equiv E[Y_i(D_i, Z_i)|D_i = D, Z_i = Z]$.

The following assumptions establish the type of variation that constitutes a valid decision-quality instrument.

Assumption A2″ (Exogeneity of D and Z). $(Y_i(0, 0), Y_i(0, 1), Y_i(1, 0), Y_i(1, 1)) \perp (D_i, Z_i)$.

Assumption A6 (Decision-quality monotonicity). For all i, $C_i(1) \geq C_i(0)$, and $E[C_i(1) - C_i(0)] > 0$.

Assumption A7 (Decision-quality exclusion restriction). For all i, Y_i^* does not depend on Z.

Assumption A2″ modifies frame exogeneity, which now requires both D_i and Z_i to be uncorrelated with confounding factors. Assumption A6

[16] A similar challenge arises in the program-evaluation context, when individuals select into treatment on the basis of the gains from doing so, as in the Roy model.

[17] A standard instrumental variable might be used to identify the effect of a treatment on choice, say $E[Y_i(1) - Y_i(0)]$, when assignment to frames is confounded with individuals' potential outcomes. In contrast, we propose to use exogenous variation in whether an individual is consistent. In other words, we are "instrumenting" for C, not for D. To map our prior results into this notation, one can interpret $Y_i(D)$ in previous sections as $Y_i(D, Z)$ for some fixed value of Z.

requires the effect of Z on consistency to be weakly monotonic for all decision makers and strictly monotonic for some. Assumption A7 requires that variation in the decision-making environment induced by Z be irrelevant from the perspective of decision makers' preferences; it ensures that Z affects behavior by altering consistency, not by changing which option decision makers prefer.[18] Like frame separability, assumption A7 does not rule out variation in Z affecting welfare by altering the real costs of choosing against the frame. Although generally untestable with the type of data we assume, assumption A7 may be tested if one observes variation in Z affecting choices in a setting without framing effects (in which case the variation in Z should not affect behavior).

Variation in Z might arise from natural experiments or be induced by researchers. For example, suppose that some decision makers were randomly assigned to a treatment group aimed at manipulating their "cognitive load"—such as by memorizing a 10-digit number—before making the decision being studied. Such experimental designs could plausibly manipulate decision makers' susceptibility to a frame (e.g., Pocheptsova et al. 2009) in ways that are unrelated to their preferences. Other examples of decision-quality instruments might include the time pressure for making a decision, the cost of obtaining or processing information about the available choices, the opportunity cost of cognitive resources at the time of decision making, the intensity of the frame (e.g., the degree to which one alternative is more salient than another), or the complexity of the choice presented (as in Brown et al. 2017).

PROPOSITION 4. Assume that assumptions A1, A3, and A4 hold at each fixed value of Z and that assumptions A2″, A6, and A7 hold. Then,

$$E\left[Y_i^* | C_i(1) > C_i(0)\right] = \frac{\bar{Y}(0,1) - \bar{Y}(0,0)}{\bar{Y}(1,0) - \bar{Y}(0,0) - (\bar{Y}(1,1) - \bar{Y}(0,1))}.$$

Proposition 4 is best understood by analogy to the identification of a local average treatment effect (LATE; see Imbens and Angrist 1994). The monotonicity assumption (A7) permits us to divide the population into three groups: the always consistent ($C_i(1) = C_i(0) = 1$), the consistency compliers $C_i((1) = 1; C_i(0) = 0)$, and the never consistent ($C_i(1) = C_i(0) = 0$). The denominator of the expression in proposition 4 measures the reduction in the size of the inconsistent subgroup as we move from $Z = 0$ to $Z = 1$, which identifies the size of the consistency-compliers group (the analog of the compliers in the standard LATE framework). The expression in the numerator measures the change in the fraction choosing $Y = 1$ under $D = 0$ as Z changes, which identifies the fraction

[18] Like frame separability, assumption A7 is about whether Y_i^* must be indexed by Z, rather than a distributional assumption.

TABLE 2
ILLUSTRATION OF PROPOSITION 4

	When $Z = 0$		When $Z = 1$	
	$Y_i(0, 0) = 0$	$Y_i(0, 0) = 1$	$Y_i(0, 1) = 0$	$Y_i(0, 1) = 1$
$Y_i(1, 0) = 0$	.35	.00		
$Y_i(1, 0) = 1$	.25	.40		
$Y_i(1, 1) = 0$			.50	.00
$Y_i(1, 1) = 1$			.05	.45
Fraction consistent	$E[C_i(0)] = .40 + .35 = .75$		$E[C_i(1)] = .50 + .45 = .95$	
Fraction consistent and prefer option 1	$E[C_i(0)Y_i^*] = .40$		$E[C_i(1)Y_i^*] = .45$	
Consistent preferences	$E[Y_i^*\|C_i(0) = 1] = .4/(.4 + .35) = .53$		$E[Y_i^*\|C_i(1) = 1] = .45/(.5 + .45) = .47$	
Fraction consistency complier	$E[C_i(1)] - E[C_i(0)] = 0.95 - 0.75 = 0.20$			
Fraction consistency complier and prefer option 1	$E[(C_i(1) - C_i(0))Y_i^*] = 0.45 - 0.40 = 0.05$			
Consistency-complier preferences	$E[Y_i^*\|C_i(1) > C_i(0)] = 0.05/0.20 = 0.25$			

of decision makers who are consistency compliers and prefer $Y_i^* = 1$. Dividing the latter by the former yields the fraction of the consistency compliers with $Y_i^* = 1$. Table 2 further illustrates the intuition behind the proposition by applying it to hypothetical data from the online-privacy example first described in the introduction. The table highlights that proposition 4 works by essentially applying proposition 1.1 separately by values of Z and then identifying $E[Y_i^*|C_i(1) > C_i(0)]$ by comparing $E[Y_i^*|C_i(0) = 1]$ to $E[Y_i^*|C_i(1) = 1]$. Unlike conventional instrumental variables analyses, the variable affected by a decision-quality instrument (consistency) is not directly observable to the researcher. Consequently, proposition 4 requires frame monotonicity in addition to the standard instrumental variables monotonicity assumption (A6) in Imbens and Angrist (1994). In addition, proposition 4 can be extended beyond binary instruments by applying the result to each pairwise combination of Z values. Such variation allows the researcher to nonparametrically trace out the relationship between consistency and preferences, similar to identification of marginal treatment effects in Heckman and Vytlacil (2007).

As with a standard instrument, the quantity identified by proposition 4 corresponds to a specific subgroup of the population—in our setting, it is those whose sensitivity to the frame varies by Z. This quantity can be of interest for several reasons. First, consider a government deciding which value of Z to implement, for example, a regulator deciding how streamlined privacy controls should be. The solution to this problem trades off the cost of selecting a value of Z that induces greater consistency against the welfare gain from doing so. The latter depends on the preferences of the decision makers who choose consistently at one candidate Z but not in another, which proposition 4 can be used to estimate. Second, proposition 4 can shed light on the underlying behavioral model. For a decision-quality instrument that affects present bias, for example, the "long-run" view of welfare in the quasi-hyperbolic discounting model described in section I predicts that all of the consistency compliers will prefer to consume the larger amount at the later date. Estimating $E[Y_i^*|C_i(1) > C_i(0)]$ can test this hypothesis. Finally, the relationship between the preferences identified by proposition 4 and the preferences identified by proposition 1 can be extrapolated to shed light on the relationship between preferences and consistency for the full population.[19] Because this extrapolation problem depends on the positive model

[19] An interesting special case occurs when all decision makers are consistent under $Z = 1$, i.e., $E[C_i(1)] = 1$. In this case, $E[Y_i^*|C_i(1) = 1] = E[Y^*]$, so choices under $Z = 1$ are a "refinement" in which the preferences of the full population are identified. Under this condition, the estimand in proposition 4 is the preferences of the inconsistent choosers at $Z = 0$, $E[Y_i^*|C_i(1) > C_i(0)] = E[Y_i^*|C_i(0) = 0]$. Consequently, one can recover the preferences of the population and of the inconsistent decision makers without further extrapolation.

generating the framing effects, we discuss it below in the context of our running examples.

1. Decision-Making Types Model of Default Effects

In this model, a decision-quality instrument exploits variation that induces a passive type to become active, or vice versa—for example, being warned about the bias before making the decision. Incorporating the decision-quality instrument into the statistical model yields $\tilde{C}_i = \beta^C \theta_i^C + \delta_i^C Z_i + \eta_i^C$ and $\tilde{Y}_i = \beta^Y \theta_i^Y + \delta_i^Y Z_i + \eta_i^Y$, where we now allow η_i^C and η_i^Y to have arbitrary correlation with each other to reflect unobserved determinants of consistency and preferences. Note that assumption A6 corresponds to $\delta_i^Y = 0 \ \forall \ i$ and assumption A7 corresponds to $\delta_i^C \geq 0 \ \forall \ i$ and $\delta_i^C > 0$ for some i. Proposition 4 sheds light on the relationship between η_i^C and η_i^Y, which, depending on the functional form of their joint distribution, can be used to recover the distribution of population preferences in the spirit of Heckman (1979; see app. sec. C.1). Again, richer variation in Z would allow one to identify the joint distribution of η_i^C and η_i^Y more flexibly; we describe one such approach in appendix section C.2.

2. Bounded-Rationality Model of Default Effects

In the bounded-rationality model, a natural source for decision-quality instruments is variation in the distribution of costs associated with choosing actively, γ_i. For example, such variation might make it easier or more difficult to select the nondefault option, perhaps by simplifying the opt-out process (expanding or reducing the number of forms to fill out or the amount of red tape). Suppose that $\gamma_i = \gamma_{0i} - \delta Z_i$, where Z_i is the binary decision-quality instrument and $\delta > 0$. In this case, the consistency compliers are those with $-\delta < |\Delta u_i| - \gamma_{0i} < 0$. Reducing γ induces this group to start choosing actively. Variation in Z thus provides information on the distribution of Δu_i, F_Δ. With a binary instrument, identifying $E[Y_i^*]$ requires imposing a functional form for F_Δ. With more variation in Z, F_Δ can be estimated more flexibly. We provide additional detail in appendix section C.3.

IV. Application to Automatic 401(k) Enrollment

In this section, we illustrate our framework with data on enrollment decisions into employer-provided 401(k) pension plans. Such plans can either be opt-in, so that new employees must actively enroll in the plan to participate, or opt-out, so that new employees are enrolled by default. A large body of research documents striking differences in enrollment and

savings behavior between opt-in and opt-out plan designs (Madrian and Shea 2001; Choi et al. 2006; Chetty et al. 2014). Unless employee saving preferences depend on whether enrollment is the default, such findings undermine the use of traditional revealed-preference analysis in this setting. In contrast, our approach accounts for the observed framing effect to shed light on employee preferences.

A. Data

Our data come from the large health care and insurance firm studied in Madrian and Shea (2001). The firm switched from an opt-in to an opt-out enrollment policy in April 1998. Under the opt-out policy, passive employees were automatically enrolled at a default contribution rate of 3% of salary. Under both designs, the employer provided a 50% match on employee contributions of up to 6%, and employee contributions into the plan were capped at 15%.

We observe whether employee i enrolls in the plan (indicated by Y_i) and whether the default is opt-in ($D_i = 0$) or opt-out ($D_i = 1$) at the date of hire. We also observe annual compensation, age, sex, and race for each employee.[20] The income, age, and racial composition of the firm's employees are typical of a large employer in the United States, although the firm's workforce is disproportionately female. Employer contributions vest in the pension after 2 years of employment. We refer readers to Madrian and Shea (2001) for additional details regarding the data and the change in plan design. As expected, participation is greater under opt-out than under opt-in: $\bar{y}_0 = 0.491$ and $\bar{y}_1 = 0.859$ (we use lowercase letters to denote estimated sample analogs to the population moments described in earlier sections).

B. Recovery of Consistent Preferences

Under assumptions A1–A4, proposition 1 allows us to identify the preferences of the consistent employees. Frame separability (assumption A1) requires that preferences over plan participation do not depend on whether the design is opt-in or opt-out. This seems likely to hold, as it is difficult to imagine that an employee's preferences over how much to save depend on how her employer chooses to structure enrollment into its sponsored retirement plan.[21] Absent assumption A1, the behavior

[20] For confidentiality purposes, we received binned data on compensation and age.

[21] Enrollment preferences could depend on the default if employees are uncertain over whether they should enroll in the plan and interpret the default as advice from their employer. However, employees in the firm we study who were not automatically enrolled did not shift their 401(k) contributions to the default contribution rate that applied to automatically enrolled employees, as would be expected if they acted on the default as advice

observed by Madrian and Shea would not constitute a framing effect, and standard revealed-preference analysis would suffice to recover employee preferences.

Frame exogeneity (assumption A2) requires that an employee's hire date (within the window studied) be uncorrelated with whether she chooses to participate under either plan design. This is the same assumption required to identify the causal effect of the change in plan design on participation; Madrian and Shea present suggestive evidence that it holds by showing that the observable characteristics of employees hired before and after the change in plan design are similar. Appendix table 1 replicates that analysis for the modified sample we study; our results are quite similar to theirs.

The consistency principle (assumption A3) requires that employees who choose to participate in the plan under both the opt-out and opt-in design actually prefer participation and, similarly, that employees who choose nonparticipation under both designs prefer not to participate. In contrast, the standard revealed-preference assumption requires that all employees—even those whose choices depend on the participation default—prefer the option that they choose. The consistency principle will be violated if employees' choices are characterized by biases that manifest themselves across both frames; for example, if employees who consistently choose not to enroll make that decision only because of present bias.[22] If employee participation decisions are biased for reasons unrelated to automatic enrollment (the observed framing effect), further deviations from revealed-preference analysis beyond the consistency principle would be needed to accurately recover preferences.

Finally, frame monotonicity (assumption A4) requires that no employee chooses to enroll when enrollment is opt-in but chooses not to enroll when enrollment is opt-out. Frame monotonicity is not directly testable without observing employees making repeated choices across multiple frames, but it can be falsified if we observe a reduction in participation rates under opt-out enrollment for any subgroup of employees. Appendix figure 2 plots employee participation by frame, for each subgroup of employees we observe. For each group, participation is greater under opt-out enrollment, consistent with frame monotonicity.

(Carroll et al. 2009). Particularly in the 401(k) context, default effects being driven primarily by trust in one's employer would be surprising, given the countervailing incentives that arise when pension plans have an employer match (Bubb and Warren 2020).

[22] Not all forms of present bias would cause the consistency principle to fail. In the models of default sensitivity studied by Carroll et al. (2009) and Bernheim, Fradkin, and Popov (2015), e.g., present bias causes individuals to procrastinate and stick with the default until they make an active choice, at which time the amount they choose to save will be optimal. Such behavior satisfies the consistency principle because those individuals who choose consistently have selected their most preferred option.

Under assumptions A1–A4, proposition 1.1 point-identifies the preferences of the consistent decision makers for enrollment. Substituting the estimated population moments into the definition of $\bar{Y}_C$ in proposition 1 yields $\bar{y}_C = 0.777$, with a standard error of 0.006.[23] Thus, under frame monotonicity, of the 63.2% of employees whose enrollment decisions are insensitive to the enrollment default, 77.6% prefer enrollment. Without frame monotonicity, proposition 1.2 implies that the fraction of consistent employees who prefer enrollment is at least 77.6%. We therefore conclude that a large majority of the consistent employees prefer to participate in the plan.

C. *Recovery of Population Preferences*

Table 3 presents the conclusions about population preferences that can be drawn from the data under assumptions of varying strength. With only assumptions A1–A3 (col. 1), the answer is not much: proposition 2 implies that one can rule out only values of $E[Y_i^*]$ below 0.350. The scope of this uncertainty is striking, given that nearly 50% of employees choose to participate even when enrollment is opt-in. Adding frame monotonicity (col. 2) allows us to tighten these bounds significantly, yielding $0.491 \leq E[Y_i^*] \leq 0.859$. Thus, under assumptions A1–A4, an observer can conclude that at least a (near-)majority of employees prefer enrollment.

Determining how large a majority prefer enrollment requires understanding the relationship between employee preferences and consistency. As a benchmark, consistency independence (col. 3) implies $E[Y_i^*|C_i = 1] = E[Y_i^*|C_i = 0]$, so that $E[Y_i^*] = 0.777$. To shed light on the plausibility of this assumption in our data, figure 1 plots estimates of $E[C_i]$ and $\bar{Y}_C$ for each demographic subgroup we observe. If consistency independence was satisfied, we would expect the slope of this relationship to be flat. Instead, the figure suggests a strongly positive relationship between preferences and consistency: groups with more consistent employees are also more likely to contain more employees who prefer participation. The slope of the estimated best-fit line is 0.78. Characterizing this result as a formal test of unconditional consistency independence requires assuming conditional consistency independence. Even so, for (unconditional) consistency independence to hold, given this finding, it would have to be the case that conditional consistency independence fails and the within-subgroup correlation between preferences and consistency exactly offsets the observed between-subgroup correlation. Because we see little reason to expect this correlation to be positive between subgroups

[23] Standard errors on $\bar{y}_C$ and other finite-sample statistics were obtained using the delta method. See sec. B of the appendix for details. Because the standard errors may converge slowly to their asymptotic limits, app. table 2 reports bootstrap-derived standard errors, which may yield better approximations in finite samples.

TABLE 3
ESTIMATES OF POPULATION PREFERENCES

	Assumptions					
	A1–A3 (1)	A1–A4 (2)	A1–A4, Consistency Independence (3)	A1–A4, Cov (Y^*, C) ≥ 0 (4)	A1–A4, Conditional Consistency Independence (5)	A1–A4, Bounded-Rationality Model (6)
Consistent employees preferring enrollment (%)	[77.6, 1]	77.6	77.6	77.6	77.6	77.6
	(76.6, 1)	(76.4, 78.9)	(76.4, 78.9)	(76.4, 78.9)	(76.4, 78.9)	(76.4, 78.9)
Inconsistent employees preferring enrollment (%)	[0, 1]	[0, 1]	77.6	[0, 77.6]	70.9	55.8
	. . .	. . .	(76.4, 78.9)	(0, 78.7)	(68.5, 73.3)	(55.1, 56.6)
Fraction of population preferring enrollment (%)	[35.0, 1]	[49.1, 85.9]	77.6	[49.1, 77.6]	75.0	69.1
	(33.9, 1)	(47.8, 87.3)	(76.4, 78.9)	(47.8, 78.7)	(73.4, 76.6)	(68.1, 70.0)

NOTE.—This table illustrates the estimation of enrollment preferences among consistent employees, inconsistent employees, and the population, under various sets of assumptions. Column 1 applies propositions 1.2 and 2.2 under our core assumptions, excluding frame monotonicity. Column 2 adds the assumption of frame monotonicity and applies propositions 1.1 and 2.1. Column 3 adds the assumption of consistency independence. Column 4 assumes, on the basis of group-level evidence in fig. 1, that the covariance between consistency and principles is positive. Column 5 assumes conditional consistency independence (proposition 3), using the same set of employee characteristics as fig. 1. Column 6 reports results from the structural estimation of a bounded-rationality model of default effects with all variation in consistency due to variation in utility stakes. All estimates are based on data from Madrian and Shea (2001) provided to the authors. We report 95% confidence intervals in parentheses under the estimates, using the method of Imbens and Manski (2004) for partially identified parameters; see app. sec. B for details. Confidence intervals for col. 6 are calculated via a nonparametric bootstrap.

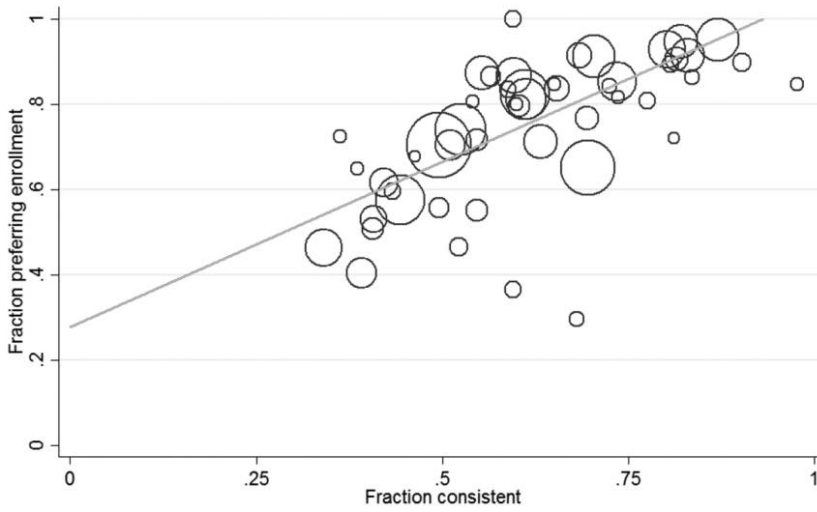

FIG. 1.—Consistency versus preference for enrollment in a 401(k) plan. Estimates are based on calculations on data from Madrian and Shea (2001) provided to the authors. Each point on the scatter plot consists of all workers with given values of compensation, age, sex, and race. The fraction consistent and the fraction of consistent decision makers preferring enrollment are calculated using the take-up rates before and after automatic enrollment in each cell. The size of the cell is proportional to the area of the circle. A color version of this figure is available online.

while being negative within subgroups, we treat this possibility as remote. Hence, we interpret figure 1 as suggestive evidence against consistency independence holding in our population.

Motivated by figure 1, an observer might feel comfortable imposing that the relationship between consistency and preferences is positive for the employees in our data, even without being confident about exactly what the relationship is. Column 4 shows the implication of this assumption for population preferences: $0.491 \leq E[Y_i^*] \leq 0.777$; roughly speaking, the assumption suggests that somewhere between one-half and three-quarters of employees prefer enrollment. If the estimated slope in figure 1 had instead been negative, the resulting bounds would be even narrower: $0.777 \leq E[Y_i^*] \leq 0.859$.

We next consider conditional consistency independence. In our setting, this assumption requires that among employees with the same income, age, gender, and race, preferences for enrollment are uncorrelated with consistency. As discussed in section III.B, this assumption is likeliest to hold when the remaining variation in employees' sensitivity to the default is mostly driven by factors exogenous to the specific decision being considered. This can be satisfied in a types model of default effects or a bounded-rationality model in which the characteristics we observe capture most of the variation in preference intensity. In both

cases, the remaining variation in preferences must not be correlated with unobserved variation in the "stickiness" of the default.[24]

Column 5 of table 3 presents the results of the matching analysis. We estimate that the fraction of inconsistent employees preferring enrollment is 70.8%—approximately 7 percentage points lower than the corresponding preferences for consistent employees. The difference in estimated preferences between the consistent and inconsistent employees is statistically significant ($p < .001$). For the full population of employees, the analysis implies that 75.0% prefer enrollment.

Finally, recall that under a bounded-rationality model of defaults, conditional consistency independence tends to fail when most of the remaining variation in consistency (after conditioning on observables) is driven by unobserved variation in the utility "stakes" of the decision across decision makers. Suppose that this is indeed the case and that decision makers behave as described by the bounded-rationality model of defaults developed above. We allow the cost of active choice to vary by demographic group. Within each group, the distribution of utility gains from enrollment are normally distributed. Thus, $\gamma_i = \gamma_X$ and $F_{\Delta|X}(\Delta u_i) \sim N(\mu_X, \sigma_X^2)$ for all i such that $X_i = X$, where X_i denotes i's group. Note that this setup represents the extreme case in which conditional consistency independence fails, since, by assumption, all of the remaining variation in consistency is driven by the stakes of the decision, $|\Delta u_i|$.[25] Within each group, observing the fraction of consistent decision makers and the preferences of that group allow us to identify μ_X/σ_X and γ_X/σ_X, which, in turn, allow us to recover the preferences of the inconsistent employees within the group. Finally, aggregating the group-specific preferences using the prevalence of each group in the population (or, using the weights

[24] For example, if cognitive ability was positively correlated with both consistency and preferences among employees of the same age, gender, race, and income, our results would yield an upwardly biased estimate for the preferences of the inconsistent employees. The bias in the matching estimator is given by $E[Y_i^*] - E_X[\bar{Y}_C(X_i)] = E_{X_i}[\mathrm{Cov}(Y_i^*, C_i | X = X_i)/E[C_i | X = X_i]]$. With the right data, one could attempt to recover population preferences by exploiting a decision-quality instrument. For example, one might randomly assign certain employees to a streamlined process for actively choosing a plan or financial counseling services to help determine whether 401(k) participation is consistent with the employee's goals for saving and retirement.

[25] In the appendix, we consider the case in which γ follows a lognormal distribution. Appendix fig. 3 illustrates how different assumptions about the relative variance between γ and Δu imply different conclusions about population preferences. When the variance of γ is relatively small, estimated population preferences approach the case in which γ is homogeneous and all the variation in consistency is driven by Δu. In contrast, when the variance of γ is relatively large, variation in consistency is driven primarily by variation in that term. And because the model assumes that γ and Δu_i are independently distributed, estimated population preferences in this case approach the estimates one obtains from assuming consistency independence. Consequently, one can interpret cols. 3 and 6 as two extreme forms of a bounded-rationality model in which the distribution of structural parameters can vary by observable group membership.

in lemma 1, the prevalence of each group among the inconsistent decision makers) allows us to recover population preferences and preferences among the inconsistent employees.

The results of this analysis are reported in column 6 of table 3. Because assumptions A1–A4 are satisfied by this model of behavior, our estimate for the preferences of the consistent employees is the same as in the other columns. In contrast, this model suggests that just 56% of the inconsistent employees prefer enrollment. Combining the consistent and inconsistent employees, we estimate that the fraction of the population preferring enrollment is approximately 69%. From this, we conclude that a literal application of conditional consistency independence is not necessary for obtaining the result that consistent employees prefer enrollment at a higher rate than do inconsistent employees.

To better understand why we estimate a positive relationship between enrollment preferences and consistency among the employees in our data, it is useful to investigate how differences in preferences and consistency relate to employee characteristics. Although we cannot directly observe either preferences or consistency for individual employees, the results in section III.B allow us to investigate differences based on employees' observable characteristics. We estimate a regression of the form

$$E[Y_i|D, X] = \alpha_0 + \alpha_1 D + X'\beta_0 + X'\beta_1 D, \tag{3}$$

where Y and D are defined as above and X is a vector of employee characteristics. Applying proposition 1 (conditional on a given realization of X) implies that

$$E[C_i|X_i = X] = 1 - \alpha_1 - X'\beta_1, \tag{4}$$

$$E[Y_i^*|C_i = 1, X_i = X] = \frac{\alpha_0 + X'\beta_0}{1 - \alpha_1 - X'\beta_1}. \tag{5}$$

The results of the analysis are reported in table 4. We find that both consistency and the preferences of consistent choosers vary systematically by employee characteristics. Variation in consistency is strongly related to variation in compensation, with those in the highest compensation bin (annual income over $50,000) 40% more likely to choose consistently than those in the lowest bin (annual income less than $20,000) and 41% more likely to prefer enrollment. After income is controlled for, preferences for enrollment, but not consistency, also vary by age, race, and gender. These findings are consistent with the nonparametric evidence of a positive relationship between consistency and consistent preferences in figure 1.

Table 4 shows that preferences for 401(k) participation are lowest among young and low-income employees (e.g., in all four groups with

TABLE 4
CONSISTENCY AND PREFERENCE BY OBSERVABLE CHARACTERISTICS

	Consistency (1)	Consistent Preferences (2)
Compensation:		
$20,000–$29,000	.123***	.197***
	(.028)	(.032)
$30,000–$39,000	.218***	.319***
	(.033)	(.033)
$40,000–$49,000	.267***	.368***
	(.035)	(.034)
>$50,000	.398***	.407***
	(.034)	(.033)
Age:		
30–39 years	−.033	.008
	(.022)	(.017)
40–64 years	.025	.068***
	(.023)	(.017)
White	−.015	.087***
	(.021)	(.016)
Male	.003	−.059***
	(.021)	(.017)
Observations	9,887	9,887

NOTE.—Estimates are based on eqq. (3)–(5), using disaggregated data from Madrian and Shea (2001) provided to the authors. The left-out groups for each demographic characteristic are (1) employees with compensation less than $20,000, (2) employees with age less than 30 years, (3) nonwhite employees, and (4) female employees. Column 1 examines the conditional probability that an employee is consistent. Column 2 examines the conditional probability that a consistent chooser prefers enrollment, holding other characteristics constant. Standard errors, calculated using the delta method, are reported in parentheses.

*** $p < .01$.

employees below age 30 and salary below $20,000, we estimate that a majority prefer nonparticipation). One explanation could be that younger and lower-income employees are more susceptible to present bias, so that the consistency principle (assumption A3) is more strongly violated for them than for other groups. The preferences revealed by consistent choices would then still contain some bias, as in example 3 of section I. Alternatively, it could be that these employees expect their tenure at the firm to be too short for the employer matching contribution to vest, in which case even a modest preference for liquidity could lead to a preference for nonenrollment. Figure 2 investigates this hypothesis by plotting preferences for enrollment by demographic group against the fraction of the group remaining at the firm 2 years from the date of hire—the time at which the 50% employer matching contribution vests. The observed relationship is strong and positive. In addition, the best-fit line for the group-level regression has an R^2 of over 90%, suggesting that our estimated preferences are a strong predictor of ultimate tenure at

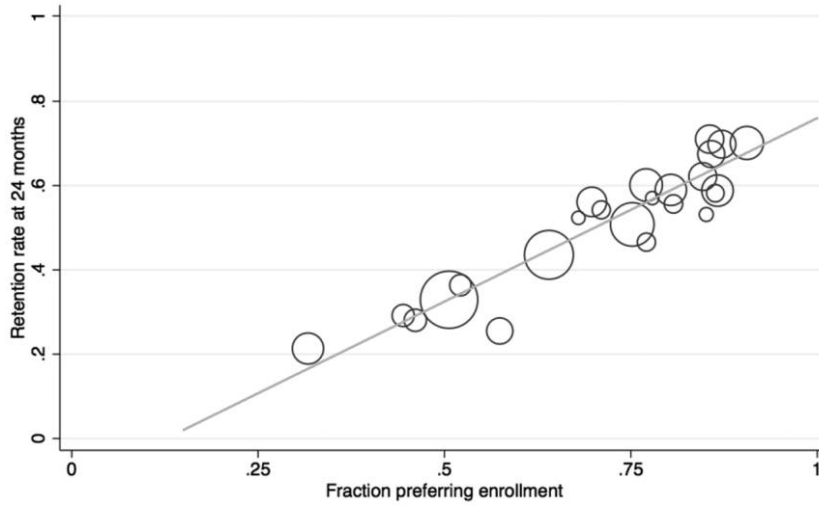

FIG. 2.—Retention rates and preferences for enrollment. Estimates are based on calculations on data from Madrian and Shea (2001) provided to the authors. We plot the retention rate at 24 months against the fraction preferring enrollment, for each observable group. Each point on the bubble scatter plot consists of all workers with given values of compensation, age, and sex. The size of the cell is proportional to the area of the circle. Our interpretation of the figure assumes conditional consistency independence; without this assumption, the *x*-axis can be interpreted as the fraction of consistent choosers preferring enrollment. Because of additional sample size restrictions necessary to study retention rates, we do not include race as a covariate. Earlier matching results are similar without using race as a covariate. Data on retention are calculated using captures of the relevant data at specific dates roughly 6 months apart. An employee is counted as retained in a month if he or she is still at the firm as of the most recent date of observation. Results are similar using retention at 12 or 18 months. A color version of this figure is available online.

the firm for most groups. This evidence is reassuring, as it is consistent with the hypothesis that the preference information recovered by our analysis is driven by rational differences in employee expectations, rather than simply reflecting unobserved biases.

D. Discussion

The above results illustrate how researchers can use our proposed framework to recover more credible estimates of preference information than what would otherwise be available. Using the type of data that is already routinely collected, we show how the preference information that can be learned depends on the strength of the assumptions an observer is willing to impose. A central virtue of this approach is its transparency—researchers who rely from the start of their analysis on a specific behavioral model often implicitly impose assumptions A1–A4 in addition to

some relationship between preferences and consistency. Thus, even if one ends up relying on a specific behavioral model, our approach highlights how the assumptions of the model contribute to identification—that is, how they narrow the identification region relative to weaker assumptions, such as assumptions A1–A4 alone.

In the specific application we study, our results show that under relatively weak assumptions (A1–A4), the vast majority (78%) of consistent employees at the firm we study prefer 401(k) enrollment, as do between 49% and 86% of all employees. Adding an assumption about the direction of the relationship between preferences and consistency based on the nonparametric evidence in figure 1 allows one to narrow these bounds to conclude that the fraction of all employees preferring enrollment is somewhere between 49% and 78%. Imposing a stronger, but still plausible, assumption in the form of conditional consistency independence allows one to point-estimate the fraction of employees preferring enrollment at 74%. Alternatively, one can pin down the relationship between preferences and consistency by imposing a specific behavioral model; the one we consider yields a lower estimate of population preferences (69%) but agrees with our matching estimator that the inconsistent employees tend to prefer enrollment at a lower rate than the consistent employees.

V. Cardinal Welfare Metrics and Price Variation

Thus far, we have focused exclusively on the identification of ordinal preferences, the object directly identified by conventional revealed-preference analysis. In this section, we consider how the preference information recovered by our approach relates to standard money-metric welfare measures. As is the case without framing effects, constructing these welfare measures requires observing variation in the relative price of the available options.

Let $p \in \mathbb{R}$ denote the price of option 1 relative to option 0. We assume that the fraction of individuals choosing option 1 is observed (or can be estimated) at any price p and under each frame $D \in \{0, 1\}$, and we denote it $\bar{Y}(p, D) = E[Y_i(p, D)]$.

Holding price fixed, this model is isomorphic to the main model considered above. Our identifying assumptions have simple analogs here, supposing that they hold at any fixed price. In that case, the results we describe can be used to identify $E[Y_i^*(p)]$ for any fixed p.[26] One can then trace out this measure of *frame-free demand* for option 1—that is, demand once framing effects have been eliminated and individuals choose

[26] We set aside the question of exactly which approach is used to identify $\bar{Y}^*(p)$.

according to $Y_i^*(p)$—at each p to obtain the *frame-free demand curve*, $\bar{Y}^*(p)$. Denote the inverse of this demand curve by $p^*(\bar{Y})$.

For simplicity, assume that decision makers have quasi-linear preferences, $U_i(Y_i, x_i) = u(Y_i) + x_i$, for some numeraire x_i, and budget constraint $pY_i + x_i = z_i$. Given quasi linearity, we can express the money-metric difference in utility between the two options as $U_i(1, z_i - p) - U_i(0, z_i) = p_i^* - p$, where p_i^* denotes the reservation price for individual i, $p_i^* \equiv u_i(1) - u_i(0)$. Social welfare is $W(p, D) = \int_i U_i(Y_i(p, D), z_i - pY_i(p, D))$.

Knowledge of $\bar{Y}^*(p)$ alone is sufficient to answer a number of questions of interest. Intuitively, the function substitutes for the standard demand curve used to estimate equivalent or compensating variation. For example, one could evaluate the welfare effect of assigning each individual with option 1 versus assigning each individual with option 0 by integrating $p^*(\bar{Y})$ over the population. As another example, one could use $\bar{Y}^*(p)$ to calculate the welfare effect on consumers of a price change that occurs under a "refinement" frame (D^*) in which all individuals choose according to $Y_i^*(p)$.

Conducting welfare comparisons when some choices are made under $D = 0$ or $D = 1$ requires additional structure because these comparisons depend on the joint distribution of $Y_i(0, p)$, $Y_i(1, p)$, and $Y_i^*(p)$. We sketch one strategy for dealing with this issue here.

Assumption A8. For each individual i and frame D, there exists a unique reservation price, $p_i(D)$, such that $p < p_i(D) \Leftrightarrow Y_i(p, D) = 1$.

Assumption A9. There exists a common index i such that $Y_i(p, 0) > Y_{i'}(p, 0) \Leftrightarrow Y_i(p, 1) > Y_{i'}(p, 1) \Leftrightarrow Y_i^*(p) > Y_{i'}^*(p)$ for any i, i'.

For example, these two assumptions would be satisfied under a model in which decision makers are consistent whenever the intensity of their preference exceeds some threshold. Given this additional structure, we are guaranteed to have three well-defined inverse demand curves: $p(\bar{Y}, 1)$, $p(\bar{Y}, 0)$, and $p^*(\bar{Y})$, corresponding (respectively) to $\bar{Y}(p, 1)$, $\bar{Y}(p, 0)$, and $\bar{Y}^*(p)$. Note that frame monotonicity implies $p(\bar{Y}, 1) \leq p^*(\bar{Y}) \leq p(\bar{Y}, 0)$ for any $\bar{Y}$.

Assumptions A8 and A9 allow us to perform welfare comparisons for most policy changes one might be interested in. These comparisons are possible because, given the common index structure (assumption A9), we know that at a given value of $\bar{Y}$, the values of the inverse demand curves $p(\bar{Y}, 1)$, $p(\bar{Y}, 0)$, and $p^*(\bar{Y})$ correspond to the respective individual reservation prices $p_i(1)$, $p_i(0)$, and p_i^* for the same individual. Thus, recovering the three aggregate demand curves also allows us to recover the joint distribution of individual demand curves that is essential to the welfare calculation. The following proposition describes two such welfare comparisons.

Proposition 5. Suppose that assumptions A1–A4 hold at any price; assumptions A8 and A9 imply

5.1. The welfare impact of an increase in price from p_0 to p_1 under the frame $D = 0$ is $W(p_1, 0) - W(p_0, 0) = -(p_1 - p_0)\bar{Y}(p_1, 0) - \int_{\bar{Y}(p_0,0)}^{\bar{Y}(p_1,0)} [p^*(\bar{Y}) - p_0]\, d\bar{Y}$.

5.2. The welfare impact of changing the frame from $D = 0$ to $D = 1$ is $W(p, 1) - W(p, 0) = \int_{\bar{Y}(p,0)}^{\bar{Y}(p,1)} [p^*(\bar{Y}) - p]\, d\bar{Y}$.

Figure 3 illustrates these welfare calculations. Figure 3*A* illustrates proposition 5.1, the welfare effect of increasing the price under $D = 0$. The price change reduces welfare by inducing some of those who were consuming option 1 to switch to option 0 and also by raising the price for those who continue to consume option 1. The welfare calculation for a price change under $D = 1$ is analogous.

Figure 3*B* illustrates proposition 5.2, the welfare effect of changing the frame from $D = 0$ to $D = 1$, at some fixed price p. The welfare effect of changing the frame falls entirely on inconsistent choosers in this model because these are the individuals who change their behavior when the frame changes.[27] The inconsistent choosers who prefer option 0 at price p are made worse off by a switch to $D = 1$ (dark-gray region in the graph) and the inconsistent choosers who prefer option 1 are made better off (light-gray region). The amount by which a given individual is better or worse off is determined by $p_i^* - p$. By integrating the estimated demand functions, one can therefore determine whether welfare is higher under $D = 0$ or $D = 1$. The figure is drawn for the case in which $D = 0$ leads to higher welfare at the specified price.

VI. Conclusion

Recovering preferences when framing effects are present is a fundamental challenge in behavioral economics. Our proposed approach is to maintain the revealed-preference assumption unless an apparent framing effect is observed. In that case, we relax the revealed-preference assumption as much as is required to accommodate the observed framing effect, but no further. We show that this transforms the original preference-recovery problem into one of accounting for potentially endogenous selection into the subpopulation of consistent decision makers. Applying this approach can lead to novel insights even in well-studied settings such as automatic enrollment into pension plans, as illustrated in the empirical application.

[27] One could incorporate a cost of choosing against the frame into individual welfare, in which case the consistent individuals would also matter for evaluating the change in welfare (see Goldin and Reck 2020).

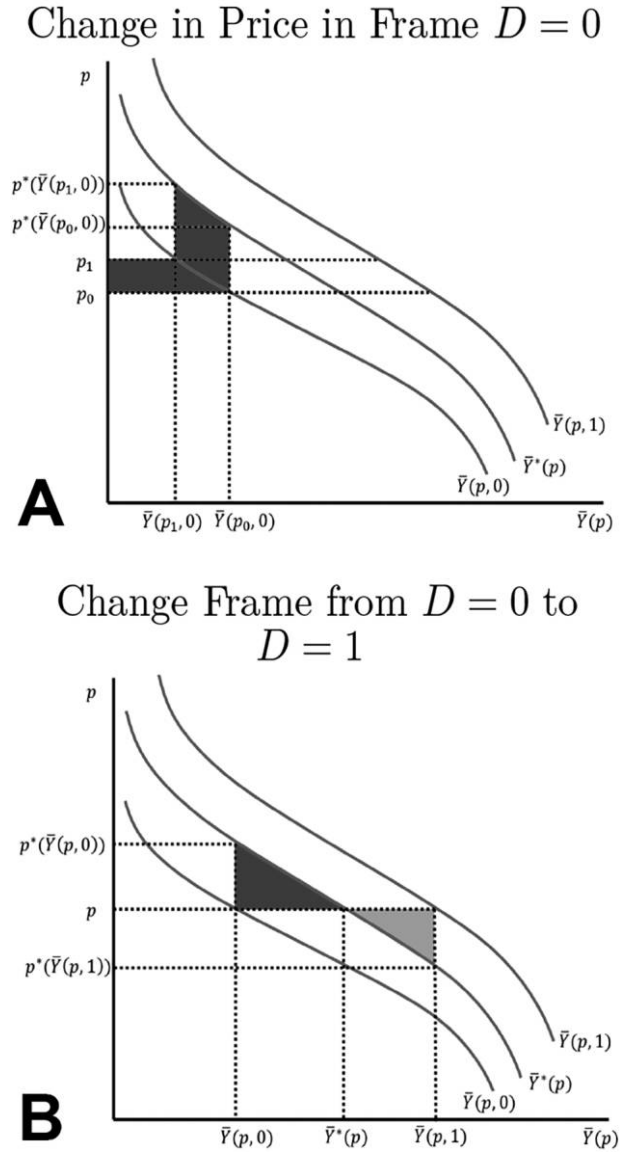

FIG. 3.—Illustration of welfare calculations. This figure illustrates the welfare effects of a change in the price under a frame $D = 0$ (A) and a change in frame from $D = 0$ to $D = 1$ at fixed price p (B). Decreases in welfare are represented as dark-gray regions; increases are represented as light-gray regions. A color version of this figure is available online.

More generally, our approach offers two main advantages to empirical researchers when framing effects are present. First, it offers simple and practical tools to recover preference information without having to commit to a specific behavioral model or set of functional form assumptions. In the many settings in which our core assumptions are plausible, propositions 1 and 2 allow the researcher to estimate the preferences of consistent decision makers and to bound the preferences of the population. Point-identifying population preferences raises additional challenges, and, depending on the application, the empirical tools we developed might offer a path forward. But even when our tools are unlikely to apply, understanding the preference-recovery challenge as a selection problem may still provide insight. For example, a researcher's knowledge of an application may suggest that preferences and consistency are positively correlated, which, in conjunction with equation (2), would narrow the range of values in which population preferences might fall.

The second benefit to our framework is that it makes model-based approaches to preference identification more transparent. As long as the model satisfies our core assumptions, our results shed light on which features of the model are driving the identification, namely, the behavioral and distributional assumptions that pin down the relationship between preferences and consistency. Highlighting these features can help researchers choose between models and assess the credibility of their results.

At the same time, our approach is subject to at least two important limitations. First, the reduced-form nature of our proposed tools might lure researchers into using them in settings in which their identifying assumptions are not met. We have tried to alleviate this concern by highlighting the types of conditions in which each tool is valid in the context of specific behavioral models.

Second, we have assumed throughout that the presence of the framing effect is the only reason that decision makers' choices fail to reflect their preferences. When other biases cause choices to diverge from preferences, choices will not reveal preferences, even once the framing effect has been removed. In such cases, applying our approach still yields the (counterfactual) choices decision makers would make if the framing effect had not been present, but the preferences inferred from those (counterfactual) choices must be further adjusted before preferences can be recovered. For example, one might combine our approach with the analysis of decisions made by experts, or some other reference group (see Handel and Schwartzstein 2018), to test whether the expert choices resemble the nonexpert choices once the influence of framing effects has been removed. If not, it might suggest that biases other than framing effects are playing a role.

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