Problem 3: Evasion Technology and the Distribution of Evasion

Why do only rich people evade taxes using offshore accounts? A popular intution about the answer to this question is that high-income people have more money to hide, so it is worth it for them to incur a given fixed cost to evade offshore. This question attempts to formalize this idea.
We begin with an Allingham and Sandmo model of tax evasion.

$$Eu = (1-p)u(\bar{y}(1-\tau)+\tau e) + pu(\bar{y}(1-\tau)-\theta\tau e)$$

1. Assume decreasing absolute risk aversion. Show that evasion is increasing in true income $\bar{y}$.

2. Let the "Tax Gap" for an individual be the fraction of income evaded $g = e/\bar{y}$. Note that with linear tax this is the same as the fraction of tax due that is evaded. Let $g_0(p, \bar{y})$ be optimal g given probability of detection p, income $\bar{y}$, and $\tau$and θ (supressed as inputs). Show that constant relative risk aversion implies that $g_0(p, \bar{y})$ is a constant function of $\bar{y}$. Note that we can re-write our original utility function as

$$Eu = (1-p)u(\bar{y}[1-\tau+\tau g]) + pu(\bar{y}[1-\tau-\theta\tau g])$$

3. Henceforth, we will assume constant relative risk aversion, i.e. $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. We can do much of this without this assumption but the proof becomes quite complex. Suppose the consumer has an option to pay a fixed cost γ to lower the probability of detection to $p_1 < p$ (e.g. by moving income into an offshore account). When the consumer pays this cost and evades some amount e, expected utility is

$$Eu = (1-p_1)u(\bar{y}(1-\tau)+\tau e-\gamma) + p_1 u(\bar{y}(1-\tau)-\theta\tau e-\gamma).$$

   If the comsumer adopts this evasion technology, how does e change? How does g change?

4. Suppose that taxpayers differ only in terms of their true income $\bar{y}$. We want to understand how adoption of the technology will vary with income. As an intermediate step, show that for sufficiently large $\bar{y}$, optimal g here converges to $g(p_1)$ from part 2 (which recall is a constant function of $\bar{y}$ under CRRA). Explain intuitively why this occurs. Hint: using a similar trick to before we can re-write expected utility as:

$$Eu = (1-p_1)u(\bar{y}[1-\tau+\tau g-\frac{\gamma}{\bar{y}}]) + p_1 u(\bar{y}[1-\tau-\theta\tau g-\frac{\gamma}{\bar{y}}])$$

5. Show that for sufficiently large y, the individual adopts the technology (Hint: decompose the difference in utility between adoption and non-adoption into a term involving the fixed cost only and a term involving the lower probability only. Then use your answer to part 4 to simplify the former. Be wary of the fact that at large $\bar{y}$, u' approaches zero).

6. This is basically a demand-side model explaining the concentration of sophisticated evasion at the top of the income distribution. Contrast this with the supply-side model in Alstadsaeter, Johannesen, and Zucman (2019). Which story do you think is ultimately more plausible? Can you devise an empirical test that distinguishes between them?